00306

AN IMPROVED HYPERELASTICITY RELATION FOR MODELING STRAIN RATE DEPENDENCY OF HIGH DAMPING RUBBER

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1. Introduction

In recent years considerable research efforts have been made for the development and application of high damping rubber (HDR) in base isolation bearings to protect structures from earthquakes. Although the general mechanical behavior of HDR is similar to the traditional natural rubbers (NR), there are some characteristic aspects in HDR response that demand special attention in its constitutive modeling. Figure 1 presents the typical uniaxial stress-stretch (i.e. 1+dL/L, where L is the undeformed length) responses obtained from HDR in comparison to NR. The comparison indicates the presence of a significant high initial stiffness and strain rate-dependency feature in HDR response than those of NR. In this context, the aim of this work is to model the rate dependent mechanical response of HDR and include its high initial stiffness feature.

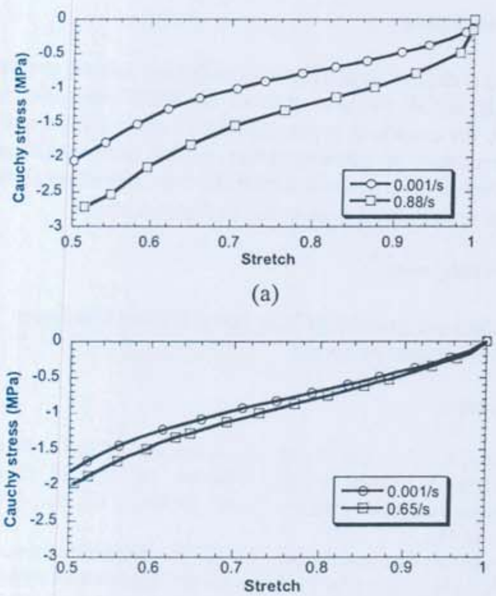

Figure 1. Responses obtained from monotonic compression test at different strain rates. (a) HDR, (b) NR

Figure 2 presents a schematic representation of typical rate-dependent responses obtained from a viscoelastic solid. When a viscoelastic solid is loaded at an infinitely slow rate, the stress-strain curve follows the E-E' path. This behavior is called the equilibrium response. On the other hand, in case of an infinitely fast loading rate, the response takes the I-I' path. Such a response is known as the instantaneous response. Both equilibrium and instantaneous responses are elastic responses and the domain of viscosity lies in between these two states. In this context, in a physically motivated rate dependent model structure, an adequate hyperelasticity law is required to describe these two elastic boundary states of the material.

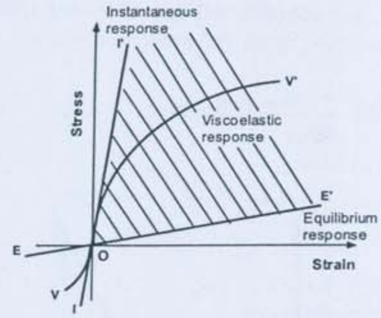

Figure 2. Typical responses from a viscoelastic solid.

In this context, over the last six decades, considerable efforts have been made to express the nonlinear elastic behavior through hyperelasticity relations. Among these relations. Mooney-Rivlin model[1] is the oldest one but only capable of representing the NR behavior up to moderate strain range. Subsequently, Hart-Smith[1], Yeoh[1] Arruda & Boyce[2] Yamashita and Kawabata[3] proposed other improved models to represent large strain response. However, none of these models can incorporate the high initial stiffness present in HDR response.

With this background, the paper presents a new hyperelasticity relation for uniaxial case to include the initial stiffness feature present in the equilibrium and instantaneous states and compares its performance with other published models. Finally, the simulation results obtained from a finite deformation rate-dependent model incorporating the new relation have been presented in comparison to experimental results to display the performance of the new relation.

2. Hyperelasticity modeling

In hyperelasticity, under the assumption of isotropy, the stress-strain relationship is derived from a strain energy density function W expressed either in terms of the strain invarients or principal stretches. In the first approach, the three strain invarients (i.e. I_1, I_2, I_3) are expressed as:

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad , \\ I_2 &= (\lambda_1\lambda_2)^2 + (\lambda_2\lambda_3)^2 + (\lambda_3\lambda_1)^2, \\ I_3 &= (\lambda_1\lambda_2\lambda_3)^2, \end{aligned} \tag{1}$$

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where $\lambda_1, \lambda_2, \lambda_3$ are the principal stretches. Under the assumption of incompressibility, I_3 reduces to unity allowing I_1 and I_2 to describe W. However, after considering the domain of I_1 and I_2 for different deformation modes over the tension and compression stretch ranges, Lambert-iani & Rey[4] concluded that only I_1 to be sufficient for modeling uniaxial response. We incorporate this conclusion to improve the existing Yamashita & Kawabata model[5] to include the initial stiffness part and thereby derive a modified hyperelasticity relation for uniaxial case. Equation 3 presents the new strain energy density relation as a function of I_1.

$$W = C_5(I_1-3) + \frac{C_3}{N+1}(I_1-3)^{N+1} + \frac{C_4}{M+1}(I_1-3)^{M+1} \qquad (2)$$

where C_5, C_3, C_4, M and N are material parameters. Figure 3 presents the performance of the proposed W relation in comparison with the conventional hyperelasticity relations in terms of Error (%) in stress (σ) prediction where,

$$\%\text{Error} = \frac{\sigma_{\text{Expt}} - \sigma_{\text{Predicted}}}{\sigma_{\text{Expt}}} \text{x}100 \qquad (3)$$

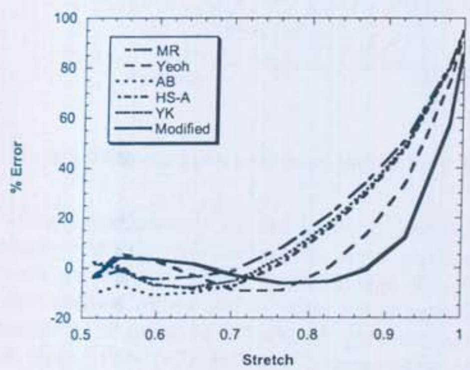

Figure 3. Comparative performance of proposed and conventional hyperelasticity relations in stress prediction in compression regime at 0.88/s strain rate.

The comparison clearly indicates the inadequacy of the conventional models in representing the stress-stretch response in low stretch level (up to 0.85) and displays the improvement achievable with the proposed relation. However, above 0.80 stretch level, all the models have shown good performance.

3. Simulation results

To reach the final goal of obtaining a rate dependent constitutive model for HDR, the hyperelasticity relation proposed in Section 2 has been incorporated in a finite deformation rate-dependent model structure and the parameters for equilibrium and instantaneous response and viscosity effect have been determined. The readers are referred to the earlier communication[5] for the details of the model structure and parameter identification scheme. Figure 4 presents the simulation results of HDR for three strain rate cases in comparison with the experimental findings. In all the cases, the model displays the capability of capturing the HDR response in low as well as high stretch levels.

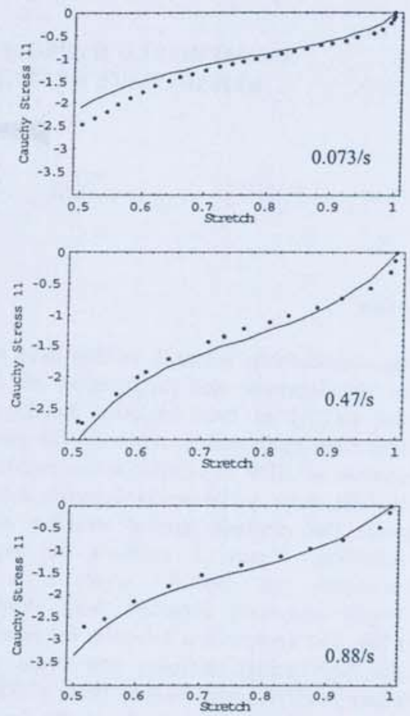

Figure 4. Simulation of monotonic compression test at different strain rates for HDR; (-) Numerical simulation, (•) Experiment.

4. Conclusion

The conventional hyperelasticity relations cannot predict the high initial stiffness present in HDR response. In contrast, the proposed hyperelasticity relation can improve this prediction. A rate-dependent model using modified hyperelasticity relation can depict the rate dependent HDR response including its high initial stiffness feature.

Acknowledgement

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